

Performance of CSI-based Multi-User MIMO for the LTE Downlink

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ABSTRACT

We consider the application of a channel state information (CSI) based multi-user (MU) multiple input multiple output (MIMO) scheme to the downlink of 3GPP Long Term Evolution (LTE) cellular networks. For that purpose, we propose a novel feedback method for providing CSI to the base stations (BSs) based upon standard vector quantization techniques. Moreover, we introduce and compare different ways for additionally signaling information about the interference situation observed by a certain user equipment (UE) to its serving BS. Apart from the actual CSI itself, the interference information is essential for choosing appropriate precoders and particularly for facilitating an efficient link adaptation. The performance of the proposed scheme is thoroughly investigated for different parameter settings by means of extensive system-level simulations and it is shown that considerable performance gains can be obtained compared to standard LTE Release 8 networks, which support channel quality indicator (CQI) feedback only.

Categories and Subject Descriptors

C.2.1 [Computer-communication networks]: Network architecture and design—*Wireless communication*

General Terms

Algorithms, Performance

Keywords

3GPP Long Term Evolution, multi-user MIMO

1. INTRODUCTION

Next generation cellular systems such as the 3GPP LTE, have to provide significantly higher spectral efficiencies compared to today's 3G networks in order to meet the ever increasing demand for higher data rates with the limited radio spectrum available for that purpose. Key technologies

included in the specification of LTE therefore comprise advanced mechanisms such as orthogonal frequency division multiple access (OFDMA), frequency-selective scheduling as well as efficient multiple-input multiple-output (MIMO) transmission schemes [1]. The latter ones generally include also so-called multi-user (MU) MIMO schemes, where multiple user equipments (UEs) might be served simultaneously on the same frequency resources by means of proper precoding techniques, thus allowing for a more efficient usage of the available spectrum. In general, however, MU-MIMO techniques require perfect channel state information (CSI) at the base station (BS) side in order to achieve full MU multiplexing gain and hence for obtaining a high system throughput [5]. It is well-known that dirty paper coding proposed by Costa achieves the sum-rate capacity of Gaussian MIMO broadcast channels [4], but the usage of this scheme in real-world systems is usually impractical due to the immense complexity involved with non-linear coding and the need for perfect CSI of all channels at the transmitter side [7]. For that reason, in recent years a couple of alternative linear precoding techniques of lower complexity have been developed, see for example [3, 12, 13].

In general, the CSI requirement at the BS side can easily be met with acceptable accuracy for time-division duplexing systems, where the same frequency band is used for both downlink and uplink and where CSI of the downlink hence can be obtained by exploiting channel reciprocity. Obtaining downlink CSI in frequency-division duplexing (FDD) systems is generally more involved and requires appropriate feedback signaling from the UEs to the associated BS. However, if in practice the feedback information is limited to a few bits only, the accuracy of the CSI becomes limited as well, thus leading to a performance degradation compared to the optimal case with perfect CSI at the BS side. For that reason, it is essential to thoroughly investigate the impact of this imperfect CSI on the system performance if such MU-MIMO schemes are to be used in practical systems.

In this paper, we propose and evaluate the application of CSI-based MU-MIMO transmission to the downlink of 3GPP LTE cellular networks operating in FDD mode, which contains CSI-based single-user (SU) MIMO transmission as a special case. For that purpose, we also address the problem how CSI as well as information about the current interference situation might be efficiently quantized by the UEs so that it can be fed back to the corresponding serving BS via a rather low-rate feedback channel. The main difference to most previous works related to MU-MIMO with finite-rate feedback (see for example [6, 11]) is that we eval-

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uate the achievable performance in a realistic interference limited LTE network, including important features such as frequency-selective proportional fair (PF) scheduling, signaling delays, a hybrid automatic repeat request (HARQ) protocol, etc. We also compare the performance to a standard LTE Release 8 system, which supports CQI feedback only, and it is shown that with the proposed CSI-based MU-MIMO scheme significant performance gains can be achieved without increasing the overall feedback load.

The paper is organized as follows. In Section 2, we outline the considered CSI-based MU-MIMO scheme in detail while the proposed feedback method and the underlying quantization method are described in Section 3. Afterwards, some system level simulation results are presented and discussed in Section 4, followed by our conclusions in Section 5.

2. CSI-BASED MU-MIMO SCHEME

We consider the downlink of a 3GPP UTRAN LTE radio network operating in FDD mode, where all BSs and UEs are equipped with M and N different antenna elements, respectively. In contrast to LTE Release 8, however, where UEs periodically send dedicated CQI reports back to their serving BS, we assume here that instead appropriate CSI reports are generated at regular intervals, containing quantized information about the estimated downlink channel as well as the current interference situation. Clearly, this represents a major shift in the feedback paradigm compared to LTE Release 8, which, however, may find its way into the further evolution of LTE towards LTE-Advanced [10].

In order to exploit the frequency selectivity of the channel, the total available bandwidth is—similar to standard CQI reporting in Release 8—subdivided into L different subbands and for each of these subbands then a separate CSI report is generated. Based on the received CSI reports, the BS performs frequency-selective PF scheduling similar to the algorithm proposed in [9] and determines the transmission mode, precoding weights, and modulation and coding schemes (MCSs) to be used by the scheduled UEs. In this regard, on every physical resource block (PRB) both SU- and MU-MIMO transmission is possible and the scheduler generally aims at maximizing the weighted sum rate for each PRB as given by the sum of the PF metrics of the various UEs to be scheduled on the same frequency resources. However, please note that even if a certain UE is scheduled on multiple PRBs, it may only use one transmission scheme and one MCS per spatial stream due to signaling constraints in the downlink, thus leading to additional restrictions in the scheduling algorithm.

In the following, we briefly outline how appropriate precoding weights for a certain UE combination are determined. Let us denote the MIMO channel of UE k on a certain subcarrier as $\mathbf{H}_k \in \mathbb{C}^{N \times M}$, the transmitted signal as $\mathbf{x}_k \in \mathbb{C}^{r_k \times 1}$, the precoding matrix as $\mathbf{F}_k \in \mathbb{C}^{M \times r_k}$, the inter-cell interference as $\mathbf{i}_k \in \mathbb{C}^{N \times 1}$, the zero mean additive white Gaussian noise with variance σ_n^2 as $\mathbf{n}_k \in \mathbb{C}^{N \times 1}$ and the number of data streams for UE k as $r_k \leq \text{rank}(\mathbf{H}_k) \leq \min(M, N)$. Then, the received signal $\mathbf{y}_k \in \mathbb{C}^{N \times 1}$ for UE k on the considered subcarrier may be expressed as

$$\mathbf{y}_k = \mathbf{H}_k \sum_{j=1}^K \mathbf{F}_j \mathbf{x}_j + \mathbf{i}_k + \mathbf{n}_k, \quad (1)$$

with K as the number of simultaneously served UEs. In

this regard, we assume that the total transmit power per subcarrier is always constrained to P_T or equivalently that

$$E \left[\left\| \sum_{k=1}^K \mathbf{F}_k \mathbf{x}_k \right\|^2 \right] \leq P_T. \quad (2)$$

For the precoding design in case of MU-MIMO, we consider so-called regularized block diagonalization (RBD) as proposed in [13]. In contrast to conventional block diagonalization [12], RBD works with arbitrary antenna configurations and numbers of UEs and is hence not limited to cases where $KN \leq M$. With RBD, the precoding matrix \mathbf{F}_k is generally factorized as

$$\mathbf{F}_k = \mathbf{F}_{k,a} \cdot \mathbf{F}_{k,b}, \quad (3)$$

where the matrix $\mathbf{F}_{k,a} \in \mathbb{C}^{M \times M}$ is appropriately chosen in order to suppress the intra-cell interference between simultaneously served UEs on the same frequency resources while $\mathbf{F}_{k,b}$ should be designed for maximizing the signal-to-interference-plus-noise ratios (SINR) of the various UEs. Extending the original RBD approach proposed in [13] by also taking the prevalent inter-cell interference into account, $\mathbf{F}_{k,a}$ generally can be calculated as

$$\mathbf{F}_{k,a} = \tilde{\mathbf{V}}_k \left(\tilde{\mathbf{\Lambda}}_k^T \tilde{\mathbf{\Lambda}}_k + \frac{KN\sigma_n^2}{P_T} \mathbf{I}_M + \frac{\bar{P}_{I,k}}{P_T} \mathbf{I}_M \right)^{-1/2}, \quad (4)$$

where $\tilde{\mathbf{V}}_k$ and $\tilde{\mathbf{\Lambda}}_k$ can be obtained from the singular value decomposition of

$$\tilde{\mathbf{H}}_k = \left[\hat{\mathbf{H}}_1^T \cdots \hat{\mathbf{H}}_{k-1}^T \hat{\mathbf{H}}_{k+1}^T \cdots \hat{\mathbf{H}}_K^T \right]^T = \tilde{\mathbf{U}}_k \tilde{\mathbf{\Lambda}}_k \tilde{\mathbf{V}}_k^H \quad (5)$$

with $\hat{\mathbf{H}}$ as the quantized channel matrix fed back from a certain UE for the subband the considered PRB belongs to. Moreover, the mean inter-cell interference power received by UE k , which can be estimated by the serving BS based on the reported interference information from UE k , is given by

$$\bar{P}_{I,k} = E \left[\|\mathbf{i}_k\|^2 \right]. \quad (6)$$

Please note that for SU-MIMO precoding no intra-cell interference has to be taken into account and consequently the matrix $\mathbf{F}_{k,a}$ can be simplified to the identity matrix. Under the assumption that the total intra-cell interference can be suppressed, the MU-MIMO channel is effectively transformed into multiple parallel independent SU-MIMO channels and the system performance may then be further improved by maximizing the SINRs of the various UEs through choosing the second precoding matrix $\mathbf{F}_{k,b}$ as follows

$$\mathbf{F}_{k,b} = \mathbf{V}_{r_k}, \quad (7)$$

where \mathbf{V}_{r_k} is made up of the eigenvectors corresponding to the r_k strongest eigenvalues of the k -th UE's effective channel

$$\hat{\mathbf{H}}_{\text{eff}} = \hat{\mathbf{H}}_k \mathbf{F}_{k,a}. \quad (8)$$

Please note that here the number of streams r_k should also be appropriately chosen by the scheduler in order to optimize the performance.

3. CSI QUANTIZATION AND FEEDBACK APPROACH

In the following, we describe the proposed quantization and feedback method to obtain CSI at the BS side in more

detail. As already mentioned before, this method represents a fundamental change compared to 3GPP LTE Release 8, where UEs feed back CQI reports only, containing precoding matrix indicators, desired MCSs as well as a rank indicator specifying the number of data streams to be transmitted. For obtaining CSI, each UE first of all estimates over the whole bandwidth the channel from its associated BS based on cell-specific reference symbols¹. In order to keep the uplink feedback limited, only one quantized channel matrix is fed back for each subband. To this end, first of all the arithmetic mean $\bar{\mathbf{H}}_s$ of all channel estimates for subband s ($s = 1, \dots, L$) is calculated as

$$\bar{\mathbf{H}}_s = \frac{1}{K_s} \sum_{k=1}^{K_s} \tilde{\mathbf{H}}_{k,s} \in \mathbb{C}^{N \times M}, \quad s = 1, \dots, L \quad (9)$$

where $\tilde{\mathbf{H}}_{k,s}$ and K_s denote the k -th estimated downlink channel within subband s and the number of estimated channels per subband, respectively. Then, we separately quantize the direction and magnitude information of

$$\mathbf{h}_s = \text{vec } \bar{\mathbf{H}}_s = [\mathbf{h}_1^T \quad \mathbf{h}_2^T \quad \dots \quad \mathbf{h}_M^T]^T \in \mathbb{C}^{N \cdot M \times 1} \quad (10)$$

in order to allow for an efficient processing and a flexible allocation of feedback bits to either type of information. The direction information is quantized by finding the codebook vector out of the considered quantization codebook with the minimum chordal distance to $\mathbf{h}_s / \|\mathbf{h}_s\|$, where the chordal distance between two vectors is generally defined as

$$d_c(\mathbf{r}_i, \mathbf{r}_j) = \sqrt{\frac{1}{2} \left(1 - (\mathbf{r}_i^H \cdot \mathbf{r}_j)^2 \right)}, \quad (11)$$

assuming that \mathbf{r}_i and \mathbf{r}_j are both unit norm vectors. Hence, the quantized channel direction can be obtained as

$$\hat{\mathbf{h}}_{\text{direc}} = \arg \min_{\mathbf{c}_m \in C} d_c \left(\frac{\mathbf{h}_s}{\|\mathbf{h}_s\|}, \mathbf{c}_m \right), \quad m = 1, \dots, 2^B \quad (12)$$

where B and $C = \{\mathbf{c}_m\}_{m=1}^{M_{\text{direc}}}$ denote the number of feedback bits for reporting the channel direction information and the codebook consisting of $M_{\text{direc}} = 2^B$ unit norm vectors, respectively. For quantizing the channel magnitude information, in contrast, a simple scalar quantizer is applied, using equidistant quantization intervals.

The construction of the channel direction codebook C is based on the well-known Linde-Buzo-Gray (LBG) algorithm according to [8], which shows a reasonable trade-off between complexity and achievable performance. In contrast to the original LBG algorithm, however, we use similar to [14] the chordal distance as distortion measure. Please note that in a practical system ideally several different codebooks would be available to adapt to the large number of possible channel conditions that may occur, including line-of-sight and non-line-of-sight scenarios or various levels of spatial correlation. These codebooks might then dynamically selected depending on the prevalent situation.

Apart from the channel direction and magnitude information, additional knowledge about the current interference

¹Please note that with a CSI-based precoding scheme at the BS side as considered here, the UEs generally have to estimate also the used precoders for being able to demodulate the transmitted signals. In this regard, we assume that additional UE-specific reference symbols are introduced during data transmissions, which are precoded with the same precoding weights as the actual data symbols.

situation observed by a certain UE is essential at the BS side, since the interference information is required not only for the proper design of the precoding matrices according to (4), but also for the link adaptation process and in particular for the selection of appropriate MCSs. For that reason, each UE also has to perform interference measurements, where we assume that each UE is only able to measure the interference level per receive antenna. Similar to the channel direction quantization approach described above, the set of measured interference covariance matrices for each subband is arithmetically averaged as follows

$$\bar{\mathbf{R}}_{ii,s} = \frac{1}{K_s} \sum_{k=1}^{K_s} \text{diag} \left(E \left[\mathbf{i}_{k,s} \mathbf{i}_{k,s}^H \right] \right), \quad s = 1, \dots, L, \quad (13)$$

where $\mathbf{i}_{k,s} \in \mathbb{C}^{N \times 1}$ denotes the k -th interference vector for subband s and $\text{diag}(\mathbf{A})$ is a diagonal matrix obtained from matrix \mathbf{A} by setting all elements except for the main diagonal equal to zero. Clearly, $\bar{\mathbf{R}}_{ii,s}$ is a diagonal matrix and therefore we employ a simple approach by quantizing the mantissa and exponent of each diagonal element separately using equidistant intervals. Assuming that the interference received at the various antenna elements is highly correlated, thus leading to almost the same interference level per receive antenna—what should be usually the case for mobile devices of small size due to closely separated antenna elements—the uplink feedback load can be reduced by quantizing only one these diagonal elements and assuming that it is the same for all of them. Moreover, another very attractive approach for significant uplink feedback load reduction with respect to interference information signaling is the quantization of the long-term interference level observed by a certain UE only, which we will also consider in the following.

4. PERFORMANCE RESULTS

We evaluate the performance of the proposed CSI-based MU-MIMO concept using a quasi-static system-level simulator for a 3GPP LTE Release 8 system, but with CSI rather than CQI feedback [1]. For that purpose, we consider a standard hexagonal grid consisting of 19 BS sites with three sectors each and we make use of the wrap around technique in order to avoid any border effects. All involved channels are generated using the 3GPP spatial channel model (SCM) and both channel estimation errors as well as a receiver noise floor are taken into account as well. Moreover, we make use of the mutual information effective SINR mapping (MIESM) for establishing the link-to-system interface [2] and the link adaptation is modeled with a realistic round-trip delay and hybrid automatic repeat request (HARQ) with incremental redundancy. The carrier frequency is set to 2.6 GHz with a system bandwidth of 10 MHz and on average there are always 10 uniformly distributed UEs in each sector. Further important system parameters are given in Table 1.

Figure 1 shows the average spectral efficiency as well as the cell-edge throughput defined as the 5th percentile of the UE throughput distribution as a function of the number of bits per subband spent for the quantization of the channel magnitude information. In this regard, we assume that all BSs and all UEs are equipped with two antenna elements each and that only the average interference levels are known at the BS side. As can be seen from Fig. 1, by increasing the accuracy of the channel magnitude quantization, the performance first of all can be gradually improved, but at a certain

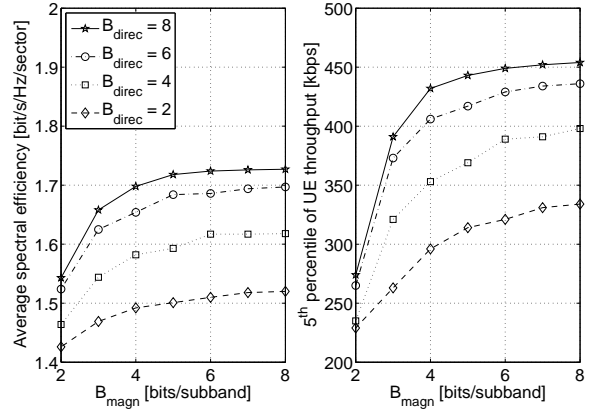
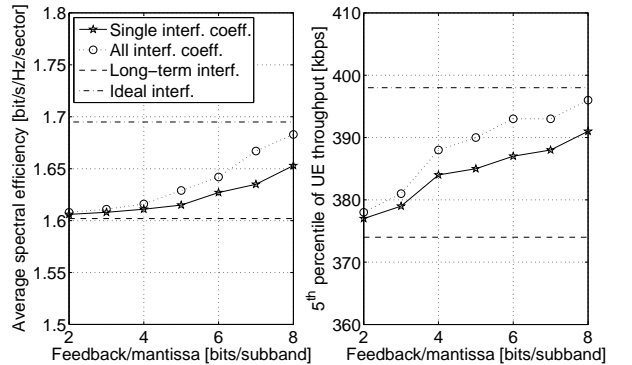
Table 1: System Level Simulation Parameters

Parameter	Setting
Deployment scenario	19 sites with 3 sectors/site
Inter-site distance	500 m
Carrier freq./ Bandwidth	2.6 GHz/10 MHz
Channel model	3GPP SCM
Shadowing standard dev.	8 dB
UE speed	3 kmph (quasi-static)
Default no. of UEs/sector	10
BS / UE antenna spacing	$10 \lambda / \frac{\lambda}{2}$, λ : wavelength
BLER target	30 %
UE / BS antennas	2 / 2 per sector
UE receiver types	MRC (single stream) / L. MMSE (dual stream)
Reporting subband size	5 PRBs
Report generation interval	5 TTIs
Feedback delay	7 TTIs
HARQ model	Synchronous, non-adaptive
Parallel HARQ processes	8
Traffic model	Full buffer services
Pilot overhead	Simulated according to [1]
Control channel overhead	First 3 OFDM symbols per subframe
Link to system interface	MIESM [2]

point the corresponding curves saturate. If, in contrast, the number of bits B_{direc} spent for the channel direction information is increased, no such saturation can be observed for the considered parameter settings. This indicates that the system performance is generally more susceptible to the accuracy of the channel direction information and therefore it seems in most cases preferable to spend more bits for the quantization of this information than to the quantization of the channel magnitudes if the total number of feedback bits is fixed.

The impact of the quantization of the interference level on the system performance is illustrated in Fig. 2. As before, we assume that all BSs and UEs have two antenna elements each and the accuracy of the channel quantization is set to $B_{\text{direc}} = 5$ bits/subband and $B_{\text{magn}} = 3$ bits/subband, respectively. Furthermore, the exponents of the various interference levels are always quantized with a resolution of 3 bits/subband. First, we note from Fig. 2 that quantizing only a single element leads to approximately the same performance as if both diagonal elements of the interference covariance matrix are quantized, thus reflecting that the two different UE antenna elements are obviously highly correlated. More importantly, however, it can be seen that the performance loss in case that only long-term interference information is available at the BS side instead of perfect knowledge is only about 5% in terms of average spectral efficiency and 6% in terms of cell-edge throughput. As a result, this method is very attractive in practice due to the small feedback load required.

In Fig. 3, we compare the performance of CSI-based SU-MIMO and MU-MIMO to a LTE Release 8 baseline system which supports only SU-MIMO transmissions with CQI feedback, assuming $M = 4$ and $N = 2$. We set the channel quantization parameters to $B_{\text{direc}} = 7$ bits/subband, $B_{\text{magn}} = 3$ bits/subband and we assume long-term interference knowledge at the BS side. This way, approximately


Figure 1: Impact of the channel feedback quantization on the system performance.

Figure 2: Impact of the feedback per mantissa of an interference coefficient on the system performance.

the same number of feedback bits per subband is spent for both feedback concepts CSI and CQI, thus leading to a fair performance comparison. In addition to the results considering realistic channel quantization, Fig. 3 shows also the upper limits of the system performance if each BS has both perfect CSI and perfect interference knowledge. It can be seen that MU-MIMO support provides significant gains in the order of 10% and 29% for the ASE and CET, respectively. In contrast to the MU-MIMO 2x2 case, however, where all BSs and UEs are equipped with two antenna elements each, not only the transmission to multiple UEs on the same resources is supported but also the transmission of multiple spatial streams to at least some of these co-served UEs is possible, which obviously results in a further performance improvement due to the higher number of degrees of freedom in that case. At this point, it should also be noted again that the performance gains due to MU-MIMO support can be achieved without the need to increase the uplink feedback load compared to the corresponding SU-MIMO case. The basically only drawback is a somewhat higher processing complexity at the BS side.

Finally, Fig. 4 shows the probabilities that a certain transmission scheme is selected for different SU- and MU-MIMO cases. In this regard, we first of all note that the SU scheme supporting dual stream transmission is considerably less of-

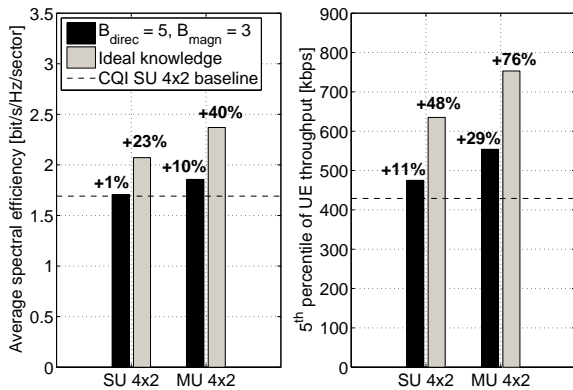


Figure 3: System performance of CSI-based MU-MIMO. The given percentages denote the relative performance gains compared to the reference case with CQI feedback.

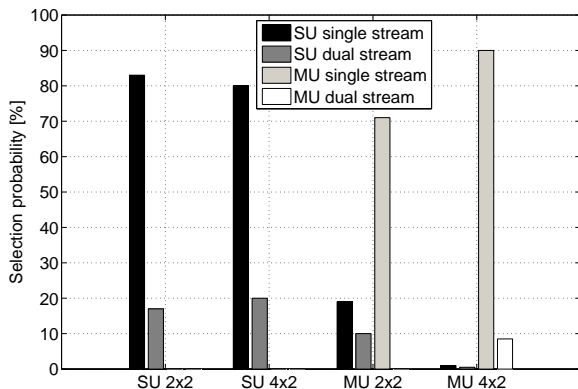


Figure 4: Transmission scheme selection probabilities for various cases.

ten chosen in case of MU-MIMO support compared to the corresponding SU-MIMO case. Furthermore, it can be seen that the probability that really multiple UEs are served on the same resources in case of MU-MIMO support significantly increases with an increasing number of antenna elements at the BS side. This is because the BS has more degrees of freedom for the precoding and therefore can better avoid interference between co-served UEs.

5. CONCLUSION

We have proposed and evaluated the application of a CSI-based MU-MIMO scheme to the downlink of 3GPP LTE Release 8 cellular networks. To this end, we have introduced an efficient feedback scheme to obtain both CSI and interference information at the base station side. By separately quantizing the channel direction and magnitude information, a flexible allocation of feedback bits is possible and the complexity of the quantization procedure can be drastically reduced. Furthermore, we considered different ways for quantizing the current interference levels and it turned out that knowledge of the long-term interference situation at the

BS side leads to merely a minor performance loss compared to the case with perfect interference knowledge. This could be explained by the fact that the interference situation is instantaneously changing between two TTIs anyway. Simulation results illustrated considerable performance gains in case of MU-MIMO support compared to the LTE Release 8 SU-MIMO reference case, assuming the same feedback load in both cases.

6. ACKNOWLEDGMENTS

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