Synchronization Requirements for OFDM-based Cellular Networks with Coordinated Base Stations: Preliminary Results

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Abstract— **Coordinating base stations is envisioned as a key component in future multi-user cellular networks. While the capacity limits of such networks are becoming well understood and promise high spectral efficiency gains, there is still little knowledge about the performance degradation that may result from imperfect synchronization among bases and mobiles. We study this problem for the downlink of base-coordinated networks that use OFDM for the air interface. Transmissions are impaired by carrier frequency offset and sampling frequency offset. We derive an accurate signal model that captures the effect of both impairments and compute the interference caused by them. We find that the carrier frequency offset requirement for base stations is an order of magnitude stricter than for mobile terminals and that the required precision can only be attained with rubidium oscillators or by high quality oven-controlled crystal oscillators stabilized by GPS.**

I. Introduction

The combination of Orthogonal Frequency Division Multiplexing (OFDM) with Multiple-Input Multiple-Output (MIMO) techniques in cellular systems with coordinated base stations (known as "network MIMO" or "Coordinated Multi-Point Transmission - CoMP") is an attractive technique for future high-capacity cellular networks [1]–[3].

It is well known that the bit error rate performance of single-user MIMO-OFDM systems is sensitive to frequency misalignments between transmitter and receiver oscillators as well as sampling timing errors ([4]). Precise synchronization is therefore vital for realizing the true potential of these kind of systems. Intuition suggests that network MIMO-OFDM systems are at least as fragile in this respect as single-user MIMO-OFDM ones. In effect, the collaborating base stations are located at different sites. Therefore, the frequency up and down converter of each one is driven by its own local oscillator. The effects of frequency misalignments among them on the system's performance (SINR loss) has not yet been studied systematically.

This work focuses on modeling the network MIMO-OFDM downlink under the combined impairments of Carrier Frequency Offset (CFO) and Sampling Frequency Offset (SFO) and study the effect of these impairments on the system performance. The case of multi-user synchronization with one base station was extensively studied in [5]. An initial analysis of the joint effect of CFO and SFO in OFDM-based space-division multiple access (SDMA) networks has been researched by the authors of this paper in [6]. That work considered all base station branches to be perfectly synchronized in carrier and sampling frequencies, and a CFO was present only at the mobiles, but the assumption is not valid for network MIMO transmission. In this study we go a step further than [6] and investigate the case when coordinated base stations are impaired with individual offsets in sampling and carrier frequency.

The remainder of this paper is organized as follows. In Section II we describe the system considered throughout the paper and derive the general signal model for a multi-user MIMO-OFDM communication under CFO and SFO impairments. In Section III we use the signal model for analyzing the network MIMO downlink and in Section IV we evaluate the effect of synchronization misalignments on the overall system performance. Conclusions are drawn in Section V.

II. System and Signal Model

We consider a cellular network with an arbitrary number of antenna branches at every base station and at every user (Figure 1). All base stations are *coherently coordinated* by means of a back-end, high-performance network which has a central processor with sufficient computational power for precoding downlink signals and decoding uplink signals in real time. We assume that it has knowledge of the entire MIMO channel matrix. Each base station and each mobile user has its own sampling and carrier frequency, within typical ranges, and there is no exchange of information between mobile users.

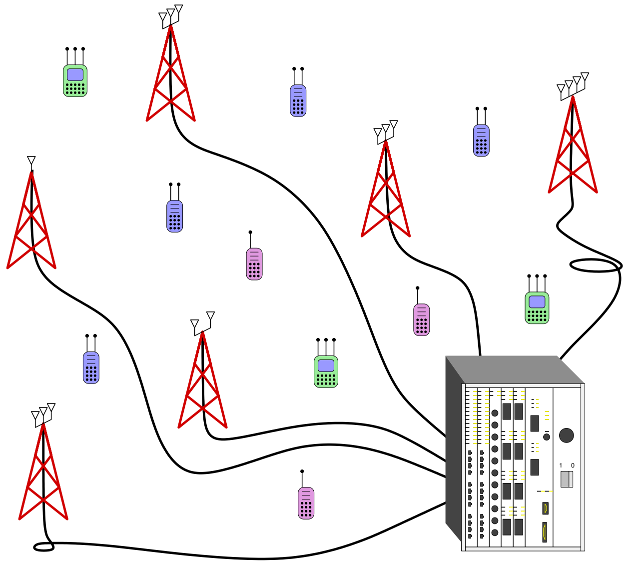

Fig. 1. Cellular network with coordinated base stations.

The network uses OFDM for the air interface. Each transmit branch, denoted by subindex i, has its individual sampling period T_i, carrier frequency f_i and respective initial phase parameters τ_i and φ_i. With some abuse of notation, the corresponding parameters are denoted T_j, f_j, τ_j and φ_j, while symbol $\jmath$ is used for $\sqrt{-1}$. The total number of transmit branches is N_t and the number of OFDM subcarriers is N_s. Subcarriers are indexed with k in the range $\left\{\frac{-N_s}{2}, \ldots, \frac{N_s}{2}-1\right\}$. The modulation of subcarrier k on transmit branch i is represented by the complex symbol $X_i(k)$. Due to possibly different sampling timings between transmitter branches at different sites, the subcarrier separation is transmit-branch-specific and measures $\delta_i = \frac{1}{N_s T_i}$ Hertz. An entire OFDM symbol is N_g samples long, equal to N_s samples plus the number of samples of the cyclic prefix. Finally, f_c is the desired (ideal) carrier frequency, n is an integer that indexes successive OFDM symbols, and $H_{ji}(f)$ is the baseband-equivalent frequency response of the passband channel between transmitter i and receiver j at frequency f. $H_{ji}(f)$ includes frequency-flat path loss and shadow fading as well as frequency-selective small-scale fading.

By following similar steps as the ones leading to equation (8) in [4] and by considering the effects of sampling by area on signal and noise, as clarified in [7], we find that the spectrum of the OFDM signal observed at any given receive branch j (could be either downlink or uplink) has the form

$$Y_j(f) = U_j(f,k) + \bar{U}_j(f,k) + N_j(f), \quad (1)$$

where $U_j(f,k)$ represents the continuous-frequency spectrum of the received multiuser signal at receive antenna j for transmitted subcarrier k and $\bar{U}_j(f,k)$ is the received spectrum of the multi-user signal of all other subcarriers $\nu \neq k$ (it is arbitrary which subcarrier is designated as k, but our analysis requires to single out one of them). They are given by

$$\begin{aligned} U_j(f,k) &= T_j \sum_{i=1}^{N_t} e^{\jmath(\varphi_j-\varphi_i)} \beta_{ji}(f,k) \\ &\quad \cdot X_i(k) H_{ji}(k\delta_i + f_i - f_c), \quad (2) \\ \bar{U}_j(f,k) &= T_j \sum_{\substack{\nu=-\frac{N_s}{2} \\ \nu\neq k}}^{\frac{N_s}{2}-1} \sum_{i=1}^{N_t} e^{\jmath(\varphi_j-\varphi_i)} \beta_{ji}(f,\nu) \\ &\quad \cdot X_i(\nu) H_{ji}(\nu\delta_i + f_i - f_c). \quad (3) \end{aligned}$$

with

$$\begin{aligned} \beta_{ji}(f,k) &= e^{-\jmath 2\pi(f_j-f_i)(nN_gT_j+\tau_j)} \\ &\quad \cdot e^{-\jmath 2\pi k\delta_i(nN_gT_i+\tau_i-nN_gT_j-\tau_j)} \\ &\quad \cdot e^{-\jmath\pi(f+f_j-f_i-k\delta_i)T_j(N_s-1)} \\ &\quad \cdot \frac{\sin[\pi(f+f_j-f_i-k\delta_i)T_jN_s]}{\sin[\pi(f+f_j-f_i-k\delta_i)T_j]}, \quad (4) \end{aligned}$$

and $N_j(f)$ is additive white Gaussian noise contributed by the front-end of receive antenna j.

The expression given by (1) through (3) makes no assumption about the synchronization among branches, and is general for any MIMO-OFDM communication (including Network MIMO). The result describes how successive OFDM symbols are distorted under constant and uncompensated CFOs and SFOs. It is valid well beyond the typical range specified for SFO in commercial OFDM systems. Note, however, that for a given SFO the FFT window of each receiver branch will eventually drift away to a point at which inter-symbol interference arises. The model stops being valid at this point. The result also assumes that all MIMO channel dispersions are shorter than the OFDM cyclic prefix. The CFO must be smaller than half of a subcarrier spacing, which implies that all receive branches of the network must have succeeded on an earlier coarse frequency acquisition stage.

It can be seen in (2) and (3) that the phase offsets due to τ_i, τ_j, φ_i and φ_j may be interpreted, without loss of generality, as part of the channel matrix. It follows that we may choose $\tau_i = 0$, $\tau_j = 0$, $\varphi_i = 0$ and $\varphi_j = 0$. We also point out that in a practical implementation of an OFDM receiver for this system, the output of the FFT corresponds to a sequence of samples of $Y_j(f)$ taken at frequencies $f = \frac{l}{N_s T_j} = \frac{l}{\delta_j}$, where l is the subcarrier index ($-\frac{N_s}{2} \leq l \leq \frac{N_s}{2} - 1$) and δ_j is the receiver-side inter-carrier spacing. Note that due to the SFOs, l points at slightly different frequencies than its peer index k at the transmit side's clock and carrier. Imposing the above conditions on (2) and (3), and focusing on an arbitrary received subcarrier $l = k$, we obtain

$$U_j(k) = \sum_{i=1}^{N_b} \beta_{ji}(k,k) X_i(k) H_{ji}(k), \tag{5}$$

and

$$\bar{U}_j(k) = \sum_{\substack{\nu=-\frac{N_s}{2} \\ \nu\neq k}}^{\frac{N_s}{2}-1} \sum_{i=1}^{N_b} \beta_{ji}(k,\nu) X_i(\nu) H_{ji}(\nu), \tag{6}$$

where

$$\begin{aligned} \beta_{ji}(k,\nu) \;=\; & e^{-\jmath 2\pi n N_g T_j (f_j - f_i)} \\ & \cdot e^{-\jmath 2\pi \nu \delta_i n N_g (T_i - T_j)} \\ & \cdot e^{-\jmath \pi \frac{N_s-1}{N_s} \left[\left(k - \nu \frac{T_j}{T_i} \right) + (f_j - f_i) T_j N_s \right]} \\ & \cdot \frac{1}{N_s} \frac{\sin\left[\pi \left(k - \nu \frac{T_j}{T_i} \right) + \pi (f_j - f_i) T_j N_s \right]}{\sin\left[\frac{\pi}{N_s} \left(k - \nu \frac{T_j}{T_i} \right) + \pi (f_j - f_i) T_j \right]}. \end{aligned} \tag{7}$$

In (5) and (6) we have approximated $H_{ji}(k\delta_i + f_i - f_c) \approx H_{ji}(k\delta_i) \stackrel{\Delta}{=} H_{ji}(k)$, because $|f_i - f_c| \ll \frac{1}{T}$ and $T_i \approx T_j \approx T$.

Considering $|f_j - f_i| \ll \frac{1}{T}$ and $T_i \approx T_j$, we may safely say that for $\nu = k$ the argument of $\sin(x)$ of the denominator in the fourth line of (7) is a small angle. Therefore, we may approximate $\frac{\sin(xN_s)}{\sin(x)} \approx N_s$ and write

$$\begin{aligned} \beta_{ji}(k,k) \;\approx\; & e^{-\jmath 2\pi n N_g [T_j (f_j - f_i) + k\delta_i (T_i - T_j)]} \\ & \cdot e^{-\jmath \pi \frac{N_s-1}{N_s} \left[k \left(1 - \frac{T_j}{T_i} \right) + (f_j - f_i) T_j N_s \right]} \end{aligned} \tag{8}$$

The approximation holds as long the OFDM symbol index n is not too large. The simulation results presented in Section IV satisfy this condition. As the exponent in the second line of (8) is also a small angle, we may join both exponentials in (8) and use $e^{\jmath x} \approx 1 + \jmath x$ to write

$$\beta_{ji}(k,k) \approx 1 + \jmath\, \theta_{ji}(k), \tag{9}$$

where

$$\begin{aligned} \theta_{ji}(k) \;=\; & -2\pi n N_g [T_j (f_j - f_i) + k\delta_i (T_i - T_j)] \\ & -\pi \frac{N_s - 1}{N_s} k \left(1 - \frac{T_j}{T_i} \right) \\ & -\pi (N_s - 1)(f_j - f_i) T_j. \end{aligned} \tag{10}$$

The above results are applied in the sequel to the Network MIMO downlink.

III. Application to the Network MIMO Downlink

We consider a Network MIMO system in which the total number of antenna branches among all bases is N_b and the total number of antenna branches of all users is $N_u \leq N_b$. For now, we assume that all bases and mobiles are equipped with one single antenna. Transmissions are precoded on each subcarrier with the pseudo-inverse of the corresponding channel matrix $\mathbf{H}(k)$, which is assumed to be perfectly known. Concretely, consider $\mathbf{S}(k)$ to be an $N_u \times 1$ vector that contains the symbols $s_u(k)$ to be transmitted to each of the N_u users on subcarrier k, and $\mathbf{W}(k)$ the $N_b \times N_u$ (tall) right-hand pseudo-inverse of the channel matrix, given by

$$\mathbf{W}(k) = \mathbf{H}^H(k) [\mathbf{H}(k) \mathbf{H}^H(k)]^{-1}, \tag{11}$$

which we assume exists. Then, the $N_b \times 1$ vector $\mathbf{X}(k)$ of symbols $X_i(k)$ to be transmitted on subcarrier k by the N_b bases is $\mathbf{X}(k) = \mathbf{W}(k)\mathbf{S}(k)$, where each element is

$$X_i(k) = \sum_{u=1}^{N_u} w_{iu}(k) s_u(k), \tag{12}$$

where $w_{iu}(k)$ are the elements of $\mathbf{W}(k)$. Because $\mathbf{W}(k)$ is the pseudo-inverse of $\mathbf{H}(k)$, it is to be noted that

$$\sum_{i=1}^{N_b} H_{ji}(k) w_{iu}(k) = \begin{cases} 1 & \text{if} \quad j = u \\ 0 & \text{if} \quad j \neq u \end{cases}. \tag{13}$$

By imposing (9), (12) and (13) on (5) we obtain

$$U_j(k) = s_j(k) + \bar{s}_j(k), \tag{14}$$

whith

$$\bar{s}_j(k) = \jmath \sum_{u=1}^{N_u} s_u(k) \sum_{i=1}^{N_b} \theta_{ji}(k) w_{iu}(k) H_{ji}(k). \tag{15}$$

Above, $\bar{s}_j(k)$ expresses the interference observed by user j due to the loss of inter-user orthogonality of the precoded transmission due to synchronization misalignments. Its effect is captured by $\theta_{ji}(k)$.

If we now impose (12) on (6) we obtain

$$\bar{U}_j(k) = \sum_{\substack{\nu=-\frac{N_s}{2} \\ \nu\neq k}}^{\frac{N_s}{2}-1} \sum_{u=1}^{N_u} s_u(\nu) \sum_{i=1}^{N_b} \beta_{ji}(k,\nu) w_{iu}(\nu) H_{ji}(\nu), \tag{16}$$

which expresses the inter-carrier interference (ICI) affecting subcarrier k of user j due to the impairments under study. It is noteworthy to point out that $\bar{U}_j(k)$ can be split into two additive components: a self-user ($u = j$) ICI and an inter-user ($u \neq j$) ICI. The first one is the well-known ICI typical of unsynchronized single-user, single-antenna OFDM systems, while the second one is an ICI enhancement due to the precoded multi-user transmission.

We may thus define a "signal to on-carrier plus inter-carrier interference ratio" (SIR) of a downlink signal received by mobile j on subcarrier k as follows

$$\mathrm{SIR}_j(k) = \frac{|s_j(k)|^2}{|\bar{s}_j(k) + \bar{U}_j(k)|^2}, \tag{17}$$

where $\bar{s}_j(k)$ is given by (15) and $\bar{U}_j(k)$ by (16). The numerical evaluation of this expression is presented in the following section.

IV. Evaluation of System Performance

For evaluating of the mobile's SIR (17) we use typical 3GPP LTE system parameters [8]. We consider $N_s = 2048$ subcarriers separated by 15 kHz. The carrier and sampling frequencies are f_c = 2.6 GHz and $\frac{1}{T} = 30.72$ MHz, respectively. The length of the cyclic prefix is 144 samples and corresponds to approximately 7% of the OFDM symbol duration. A ring of $N_b = 7$ base stations with an inter-site distance of 500 m transmits jointly to $N_u = 7$ terminals using omni-directional antennas. Out-of-cluster interference is ignored.

We model the radio channel by considering frequency-flat path loss with exponent 3.2 and shadow fading with 8 dB standard deviation and independent realizations between mobiles and base stations. Small-scale fading is modeled with impulse responses of independent circularly symmetric Gaussian distributed multipath components with an exponential power delay profile whose decay rate is $\frac{1}{2\tau_{\mathrm{RMS}}}$, where τ_{RMS} is the RMS delay spread of the channel. A rule of thumb is to choose the cyclic prefix length to be about 8 times longer than τ_{RMS}. As the cyclic prefix length is given, we set τ_{RMS} to one eight of that value. The impulse responses were limited to the length of the cyclic prefix. Independent realizations of the above model with users positioned randomly within the 7-cell environment were generated for building each instance of the network MIMO channel matrix. Perfect channel knowledge was assumed at the base stations.

The SIR given by (17) was computed per subcarrier for various cases of CFO and SFO. These are assigned randomly and individually to base stations and mobiles, with normal distribution, zero mean and various (given) standard deviations. These deviations represent the RMS dispersion of the residual CFO or SFO at each base or mobile during tracking. CFOs are specified directly as an RMS accuracy of the oscillator (for instance, a CFO RMS accuracy of 10^{-11} corresponds to 0.026 Hz for a carrier at 2.6 GHz). The contribution of SFO to the SIR is significantly weaker than the one due to CFO. Therefore, for brevity, the sequel only reports on CFO results.

For each CFO case considered, the SIR values obtained from (17) for all subcarriers and all users in 50 Network MIMO realizations ($50 \times 7 \times 2048 \approx 716000$ values per curve) are plotted in Figures 2 and 3 as complementary cumulative distribution functions (CCDF).

We assume that the base station oscillators are free running with respect to the Network MIMO system. Their carrier frequencies and sample times may or may not be stabilized by a GPS signal or any other sort of beacon, but their variation over time is assumed to be slow enough to be considered static with respect to the OFDM signaling timescale. With free running base station-side oscillators, their relative CFOs cause that the signals transmitted by the stations drift away in phase from each other as the OFDM symbol index n grows. In the absence of a mobile-side compensation technique for the multiple base station CFOs, the drift is only reset to zero when the precoder weights are updated. In LTE-Advanced this is envisioned to happen at best approximately every 7 ms, which corresponds to 100 OFDM symbols. We find that in order to serve 90% of the subcarriers (or users) for a duration of 100 OFDM symbols with an SIR due to CFO not smaller than 30 dB, the base station oscillators must have an RMS accuracy of $3 \cdot 10^{-11}$ or higher (Figure 2). This level of accuracy is only provided by free-running rubidium oscillators or high-quality oven-controlled crystal oscillators stabilized by GPS signals [9].

For mobiles, it is assumed that precoded pilot symbols are transmitted every 7 OFDM symbols (as in LTE) and used for tracking. If no corrections are done between these piloting instances, then the residues of CFO left by the last correction accrue as subcarrier phase drift over symbol index n until $n = 7$. For this reason, we provide worst-case curves when tracking is performed every 7 symbols, as well as best-case curves if CFO is corrected on every single OFDM symbol. At $n = 7$ we find that each subcarrier enjoys an SIR larger than 30 dB during 90% of the time if the terminal's tracking algorithm attains a residual RMS CFO not larger than $3.5 \cdot 10^{-4}$ of an inter-carrier spacing. (Figure 3).

By comparing the curves for $n = 1$ in Figures 2 and 3, we find that the carrier frequency synchronization accuracy requirement on the base stations is an order of magnitude higher than the one for mobile terminals.

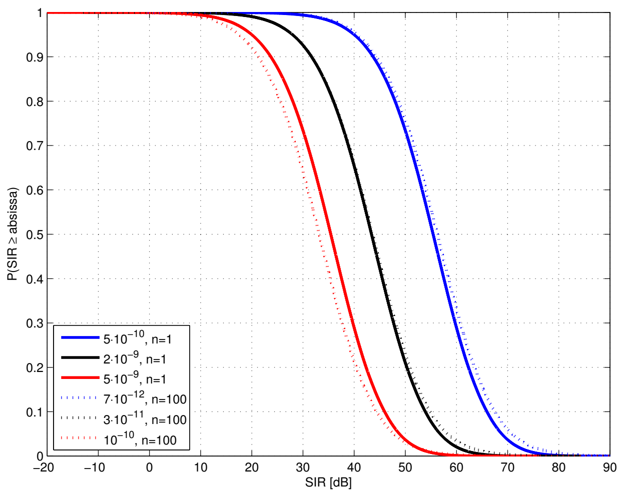

Fig. 2. SIR due to base station-side CFO for the first and 100th OFDM symbols (mobile terminals synchronized perfectly).

V. Summary and Conclusions

An exact signal model of the spectrum of received signals in OFDM-based Network MIMO systems was derived for transmissions impaired with individual carrier and sampling frequency offsets on every transmitter and receiver branch. For the downlink case, where transmissions are precoded with the inverse of the channel matrix, the synchronization mismatches cause inter-user and inter-carrier interferences. The resulting signal-to-interference ratio observed by each mobile on every subcarrier was computed numerically using the derived signal model with typical LTE parameters. The results indicate that serving 90% of the users with a SIR larger than 30 dB requires base station oscillator accuracies achievable only with rubidium oscillators or oven-controlled crystal oscillators stabilized by GPS. Mobile-side tracking algorithms must yield RMS CFO residues smaller than $3.5 \cdot 10^{-4}$ of one inter-carrier spacing in order to achieve the same performance. The accuracy specification for mobile terminals is an order of magnitude larger than for base stations.

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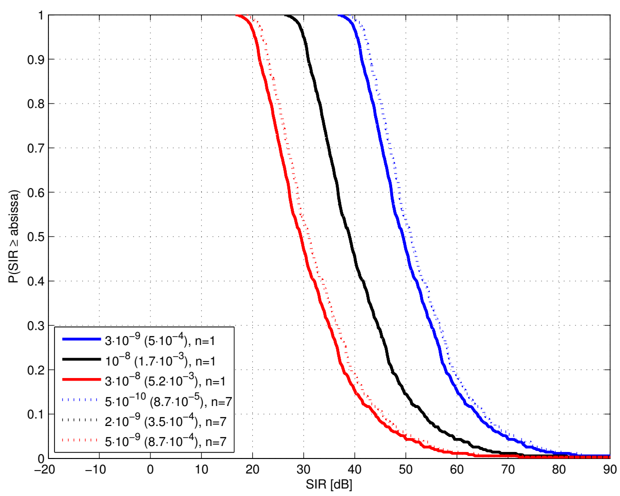

Fig. 3. SIR due to mobile-side CFO for the first and seventh OFDM symbols (base stations synchronized perfectly). The legend shows the accuracy specification in proportion to the inter-carrier spacing in parentheses.