Multi-Cell Channel Estimation using Virtual Pilots

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***Abstract*—Multicellular radio systems are often limited due to the presence of cochannel interference. Proposed physical layer concepts, e.g. coordinated joint transmission and interference rejection combining, try to strengthen the signal while combating the interference. However, the performance may be limited by the available channel knowledge. We provide a concept for multi-cell channel estimation in the downlink applicable for for both physical layer concepts. This concept uses virtual pilots based on block-orthogonal sequences, e.g. Hadamard.**

***Index Terms*—OFDMA, multi-cell channel estimation, virtual pilots, cochannel interference, interference suppression, LTE**

I. Introduction

Multi-cell interference is undoubtedly the major limiting factor in full-coverage broadband wireless access networks. Mitigating it's effect onto the downlink data transmission is one of the key challenges in future wireless communication systems. The better the knowledge about the interference channels, the better is also the basis for any interference mitigation. Multi-cell channel knowledge can be exploited by simple, receiver-based schemes as optimal interference rejection combining [1], [2] or advanced schemes based on cooperative base stations such as joint transmission [3], [4].

In order to estimate the channels of interferers, the base stations (BSs) must transmit training sequences being orthogonal among the antennas of different sectors and sites. On the other hand, quite a lot of these channels must be estimated to combat the interference until the noise floor is reached. The more interferer channels are distinguishable, the more orthogonal pilots must be transmitted. This consumes a large fraction of the potential capacity gain.

In this paper we propose a trade-off between the ability to track the interference channels and the mobility of the user. In fact, our virtual pilot scheme does not consume any more pilots than in current systems. But it enables mobile terminals to distinguish the more of the strong interference channels the slower they are moved in the service area. Hence, without increasing the pilot overhead, low-mobility terminals can take most benefit of advanced interference mitigation schemes.

The proposal is based on the current draft specification of the 3G Long Term Evolution (3G-LTE) specifying orthogonal pilot signals between different sectors while different sites are not orthogonal but marked with a pseudo-random scrambling sequence (PRS) defined by the network operator. We propose to use not random but deterministic sequences denoted as virtual pilots and provide a pilot reuse scheme based on the partial correlation principle. In this way, channels of nearby base stations can be separated using a short correlation window, e.g. over two transmission time intervals (TTIs), while estimating the channels of more distant stations requires a longer correlation window (and hence a reduced user mobility).

The paper is organized as follows. Section II describes the system model being the basis of this work. In sections III and IV, the covariance estimator as well as the correlation-based estimator and their requirements are described. Furthermore, the use of virtual pilots is suggested for multi-cell channel estimation. Results are given in section V and the paper concludes in section VI.

II. System Model

The orthogonal frequency division multiple access (OFDMA) transmission for each subcarrier Ω is described by

$$\mathbf{y}(\Omega) = \mathbf{H}(\Omega)\mathbf{x}(\Omega) + \mathbf{n}(\Omega) \ , \quad (1)$$

where $\mathbf{H}$ indicates the channel matrix; $\mathbf{y}$, $\mathbf{x}$ and $\mathbf{n}$ denote the vector of the received signal, the transmitted data symbols and the additive white Gaussian noise.

In the following, we drop the frequency index Ω and consider the multi-cellular multiple-input multiple-output (MIMO) channels in the downlink with N_T transmit and N_R receive antennas, respectively. Thus, the multi cellular transmission system may be given by

$$\mathbf{y}^m = \mathbf{h}^m_{i,u} x_{i,u} + \underbrace{\sum_{\substack{l=1 \\ l \neq u}}^{N_T} \mathbf{h}^m_{i,l} x_{i,l} + \sum_{\substack{\forall k,l \\ k \neq i}} \mathbf{h}^m_{k,l} x_{k,l} + \mathbf{n}}_{\mathbf{z}_{i,u}} \ , \quad (2)$$

where $\mathbf{y}^m$ is the received signal belonging to the m-th mobile terminal (MT) and the selected data stream transmitted from the u-th transmit antenna of the i-th BS but distorted by the signals transmitted from all other BS antennas in the system $\mathbf{z}_{i,u}$.

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III. Covariance estimation

The most practical way to perform interference reduction is directly at the MT where reliable channel knowledge is available and the interference may be detected. For interference suppression at the MT we require to obtain the system's covariance matrix $\mathbf{R}_{xx}$ of the signal, interference and noise, where $\sigma^2_{x_{k,l}}$ and σ^2_n are the power of each signal $x_{k,l}$ and the noise, respectively.

$$\mathbf{R}_{yy} = \sum_{\forall k} \sigma^2_{x_{k,l}} \mathbf{h}_{k,l} \mathbf{h}^H_{k,l} + \sigma^2_n \mathbf{I} \tag{3}$$

A technique utilizing these estimates for the purpose of interference reduction is the minimum mean square error (MMSE) receiver [2]. By employing multiple receive antennas, it is possible to use this linear receiver technique and thus enhance the desired signal while suppressing the interference. For proper application it is necessary to know the system's covariance matrix $\mathbf{R}_{yy} = \mathbf{Z}_i + \sigma^2_{x_{u,i}} \mathbf{h}_{u,i} \mathbf{h}^H_{u,i}$, defined as

$$\mathbf{R}_{yy} = \sigma^2_n \mathbf{I} + \sum_{\forall k,l} \sigma_{x_{k,l}} \mathbf{h}_{k,l} \mathbf{h}^H_{k,l} \tag{4}$$

$$\mathbf{w}^{\mathrm{MMSE}}_i = \sigma_{x_{i,u}} \mathbf{R}^{-1}_{yy} \mathbf{h}_i \tag{5}$$

Using Jensen's inequality for convex functions leads to the lower bound of the instantaneous signal to interference and noise ratio (SINR).

$$\mathrm{SINR}_i \geq \sigma_{x_{i,u}} \frac{\mathbf{w}^H_i \mathbf{h}_i \mathbf{h}^H_i \mathbf{w}_i}{\mathbf{w}^H_i \mathbf{Z}_i \mathbf{w}_i} \tag{6}$$

(6) is individually maximized by using the MMSE receiver (5) [1]. Hence, the achievable SINR with full channel knowledge for all interfering BSs is given by [5]

$$\mathrm{SINR}_i \geq \frac{\sigma_{x_{i,u}} \mathbf{h}^H_i \mathbf{R}^{-1}_{yy} \mathbf{h}_i}{1 - \sigma_{x_{i,u}} \mathbf{h}^H_i \mathbf{R}^{-1}_{yy} \mathbf{h}_i} \tag{7}$$

A. Covariance Estimator

One may think of two simple mechanisms to estimate the desired matrix. On the one hand it is possible to obtain this knowledge by estimating the covariance matrix $\mathbf{R}_{yy} = \mathrm{E}\left[\mathbf{y}\mathbf{y}^H\right]$ of the received signal vector $\mathbf{y}$ using several subsequently received data symbols. The Hermitean transpose and expectation operators are denoted by $(\cdot)^H$ and $\mathrm{E}[\cdot]$, respectively.
By assuming a transmission of i.i.d. data symbols x_k over channel k and averaging over s symbols the estimation error will decrease with s [6], [7]. The total number of transmitted data symbols across a quasi-static channel is given by S. $\widetilde{\mathbf{R}}_{yy}$ denotes the estimated covariance matrix.

$$\widetilde{\mathbf{R}}_{yy} = \frac{1}{S} \sum_{\forall s} \mathrm{E}\left[\left(\sum_{\forall k,l} \mathbf{h}_{k,l} x_{k,l}(s) + \mathbf{n}\right) \left(\sum_{\forall k,l} \mathbf{h}_{k,l} x_{k,l}(s) + \mathbf{n}\right)^H\right] \tag{8}$$

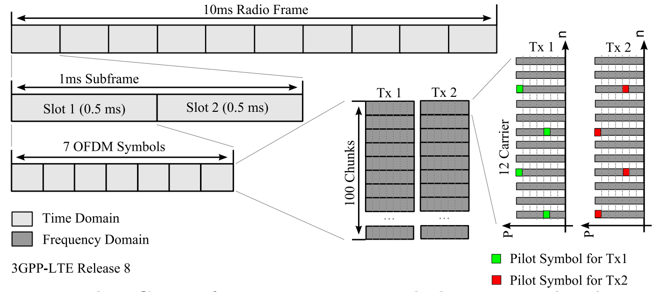


Fig. 1. The 3G-LTE framing structure including reserved orthogonal pilot symbols for intra-cell channel estimation.

The estimation error between $\mathbf{R}_{yy}$ and the estimated covariance matrix $\widetilde{\mathbf{R}}_{yy}$ is investigated in section V.

IV. Multi-cell channel estimation

Furthermore, it has been recently proposed to use coordinated joint transmission (JT) for all base stations in a service area to reduce the cochannel interference (CCI). In [3] the authors state that JT would completely remove the CCI. The ultimate capacity limit of cellular systems, given by the mean capacity of isolated cells, can be reached [4]. In practice, however, it is difficult to obtain the precise downlink channel state information (CSI) required for JT at the base station. It is shown in [8] that requirements are stricter than those for joint detection in the opposite uplink direction. Unfortunately modern cellular systems may not provide as many orthogonal pilot symbols as needed to estimate all channels independently.

A. Pilots in 3G-LTE

In the 3G-LTE specification[1], pilot symbols are intended for the purpose of intra-cell channel estimation, as indicated in Fig. 1. Zadoff-Chu sequences of length three enable the separate estimation of all three channels belonging to the sectors of the same BS. Furthermore, pilot symbols for the different transmit antennas are defined and located on different time and frequency positions, i.e. orthogonal, in a resource block. In addition, pilots are scrambled using pseudo-random sequences to be defined by the network operators.

B. Virtual pilots based on block-orthogonal sequences

Here we suggest to apply Hadamard sequences spread over the time domain from slot to slot with a maximum sequence length of 16. Fig. 2 visualizes the suggested pilot grid, where the number (hex-base) indicates the code $\mathbf{c}_\nu$ chosen from the sequence matrix $\mathbf{C}$, i.e. the row of the Hadamard matrix of maximum length $N = 16$. The decimal numbers indicate the sequence length over the time domain. Note that the suggested scheme covers different sequence length $n = \{1, 2, 4, 8, 16\}$, since the sequence pattern repeats itself every n rows. Thus the system may benefit from a more precise channel estimation for increasing sequence length n.

[1] 3GPP TS 36.211 Release 8

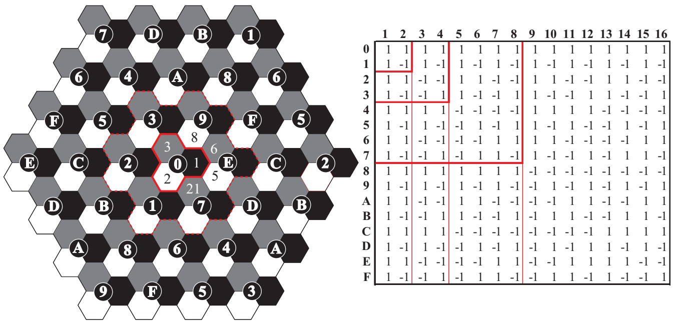


Fig. 2. Left: Pilot reuse pattern based on orthogonal code sequence, e.g. Hadamard, in a 3-fold sectorized cellular system. Decimal numbers indicate the sector index. Right: Hadamard sequences spread over space (rows) and time (columns) domain. Hex-base numbers indicate sites with the same virtual pilot sequence.

Definition 4.1 (block-orthogonal sequence): Each row of a block-orthogonal sequence matrix is orthogonal to all other rows of the same matrix with full correlation length, i.e. $\mathbf{C}\mathbf{C}^H = \mathbf{I}$. Reducing the correlation length to n yields to a matrix with block wise orthogonal properties, where each block is of size $n \times n$. Furthermore, each n-th row should be identical for a given correlation length n.

The suggested scheme assigning virtual pilot sequences to the multi-cell system is translational invariant with respect to the estimation error. Its block-orthogonality is sustained even after a cyclic shift. Note that the suggested scheme can be easily extended to the case of larger correlation length. Furthermore, each block-orthogonal sequence may be applied instead of Hadamard sequences yielding the same performance. E.g. resorting the columns and rows of the discrete Fourier transform (DFT) matrix such that block-orthogonality is given, leads to sequences having the same properties required by the correlation-based estimator. In this way, the virtual pilots can be interpreted as discrete frequency shifts by a fraction of a subcarrier spacing and partial correlation is a filtering process with limited spectral resolution.

C. Properties of the Scheme

The scheme in Fig. 2 maximizes the distance between cells using the same Hadamard sequence. After 4 cells in a row the same sequence is assigned. That applies to the horizontal and both diagonal alignments. All cells in a radius of 4 have orthogonal sequences to the cell in the middle of the scheme. The assignment of the Hadamard sequences to cells is completely defined by an arbitrary rhombus containing 16 cells each one using another pilot sequence. The rhombus is repeated to fill an infinity plane. One possible rhombus in Fig. 2 is enclosed by the cells E,0,1,7. Note that each permutation of the assignment would effect the channel estimation mean square error (MSE). In our suggested scheme it is guaranteed that the mean channel estimation error of a MT is independent of the cell where it is placed. The proof can be found in appendix A. That means it is sufficient to consider only cell 0, refer Fig. 2, for the simulation.

TABLE I
SIMULATION ASSUMPTIONS.

parameter	value
channel model	3GPP SCME[2]
type	Monte Carlo, 5000 user drops
scenario	urban-macro
additional modifications	scenario-mix[3]
f_c	2 GHz
intersite distance	500m
number of BSs	37 having 3 sectors each
BS height	32m
MT height	2m
SNR	∞

D. Correlation-based estimator

At the MT, a correlation-based estimator is used to separate the channels $\mathbf{h}_{k,l}$ for the n distinct groups. The main reason to use the correlation-based estimator is its moderate computational complexity. The correlation-based estimator is given by

$$\widetilde{\mathbf{h}}_\nu = \frac{1}{n}\sum_{p=0}^{n-1} c_\nu^*(p)\mathbf{y}^m(p)\ , \ \text{with } \nu = \{0,...,n-1\} \qquad (9)$$

where $c_\nu(p)$ and $\mathbf{y}^m(p)$ denote the code symbol and the received signal at the given discrete time index p, respectively.

V. Performance Evaluation

We apply the 3GPP SCME channel model generating single-input single-output (SISO) channels. As indicated in the parameter list, Table I, static channels are assumed for the analysis, except stated otherwise. For the sectorization, the simulation scenario is initialized cell-wise, i.e. independently for each BS. The large-scale parameters are kept fixed for all 3 sectors belonging to the same BS while the small scale parameters are randomized as indicated in [9]. A so-called scenario-mix is introduced yielding different channel states for the BSs, e.g. line of sight (LOS) or non line of sight (NLOS), which seems to be more realistic than assuming same conditions for all channels. The state is changed within the simulation following a distance-dependent stochastic process based on experimental results [10].

A. Mean Normalized Mean Square Error

For comparison of the different estimation errors, we use the mean normalized MSE, where $\mathrm{Tr}(\cdot)$ denotes the trace operator.

$$\mathrm{MSE} = \frac{\sum_{\forall n} \mathrm{Tr}\left\{\mathrm{E}\left[(\mathbf{R}_{yy}(n) - \widetilde{\mathbf{R}}_{yy}(n))(\mathbf{R}_{yy}(n) - \widetilde{\mathbf{R}}_{yy}(n))^H\right]\right\}}{N \cdot \mathrm{Tr}\left\{\mathrm{E}\left[\mathbf{R}_{yy}(n)\mathbf{R}_{yy}^H(n)\right]\right\}} \qquad (10)$$

[2]Spatial Channel Model Extended.

[3]i.e. each cell, consisting of 3 sectors, may have different channel conditions, e.g. LOS or NLOS.

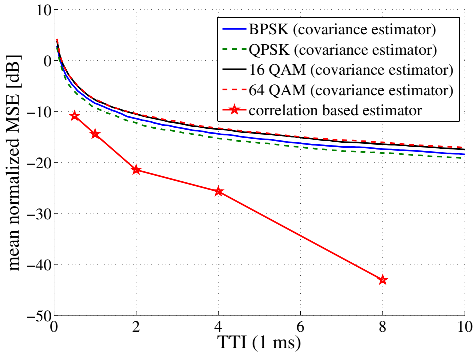
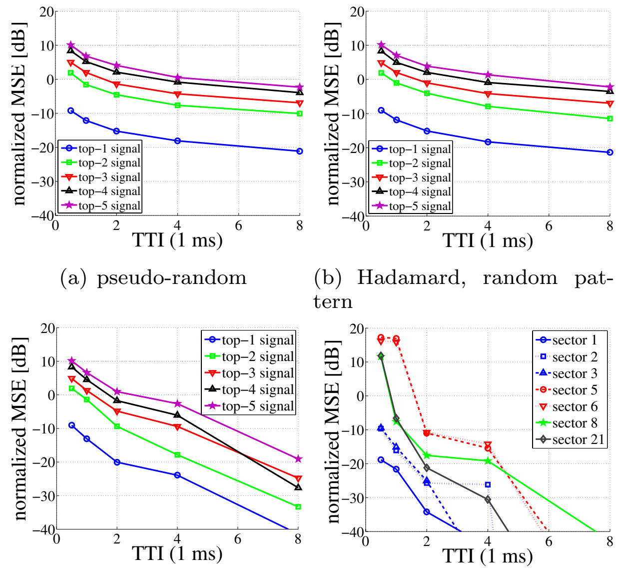


Fig. 3. Mean normalized MSE for both estimation techniques, i.e. received signal covariance estimation and multi-cell channel estimation.

Results for the covariance estimation are shown in Fig. 3. It compares the mean normalized MSE for the covariance and correlation-based estimators, applicable over all OFDM data symbols or using virtual pilots, respectively. Note that the correlation-based estimator requires at least a number of 7 OFDM symbols to be transmitted, i.e. one correlation length, where as the covariance estimator is capable to start the estimation with the first transmitted data symbol. It turns out that the correlation-based estimator outperforms the covariance estimator already for correlation length of one slot.

Fig. 4 shows the MSE, normalized by the receive power of the associated sector. It compares the different performance in the channel estimation process using virtual pilots based on pseudo-random (Fig. 4(a)), randomly arranged Hadamard (Fig. 4(b)) and Hadamard sequences (Fig. 4(c)) arranged in the specific pattern shown in Fig. 2. In these figures, the achievable MSE is given for top-N strongest sectors showing instantaneously the five highest receive powers at the MT. It turns out that using virtual pilots based on randomly arranged orthogonal sequences, e.g. Hadamard (Fig. 4(b)), cannot reduce the MSE compared to the case of using pseudo-random sequences. However, the suggested sequence reuse pattern assigning Hadamard sequences to the BSs does clearly show a superior performance compared to the random arrangement of sequences. Within that scheme BSs being closely located to each other are assigned to orthogonal sequences requiring smaller correlation lengths to be separable.

Fig. 4(c) and 4(d) indicate the achievable MSE for the top-N strongest sectors as well as for a fixed set of sectors. For the latter, we can observe that the error of the multi-cell channel estimation is less than -10 dB for sequence length larger than 4, i.e. 2 TTIs, for all sectors with index $= \{1, 2, 3, 8, 21\}$; index is indicated in Fig. 2. After full correlation length, the channel estimation of adjacent BSs is almost perfect, i.e. with a SINR > 40dB. However, estimating the top-N strongest channels may cause higher errors. This is due to the fact that the top-N strongest channels may not belong to the adjacent BSs.



(a) pseudo-random (b) Hadamard, random pattern

(c) Hadamard, fixed pattern (d) Hadamard, fixed pattern

Fig. 4. Normalized MSE obtained for the correlation estimator, in case of a static channel, for the five strongest (a,b,c) and inner sectors (d).

Nevertheless, by using a more orthogonal grid for larger correlation length it is possible to reduce the MSE as indicated in Fig. 4(c). Note that the reduction of the MSE for the strongest signals is significant. For correlation length of 16, i.e. 8 TTIs, the MSE is below -19 dB for the top-5 strongest channels.

For quasi-static channels, the suggested multi-cell channel estimation approach may be implemented easily. On the other hand, sites must be synchronized, e.g. by using global positioning system (GPS). Furthermore, phase noise must be reduced compared to present cellular systems.

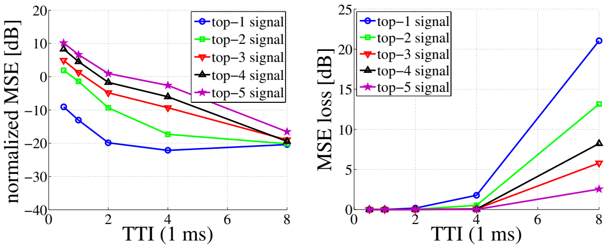
B. Time selective channels

The following part evaluates the performance degradation in the estimation process due to time varying channels, e.g. caused by a certain velocity of the MT or simply phase noise. Introducing a constant phase rotation to a static channel would result in the simplest form of a time varying channel [11].

$$\mathbf{y}^m(p) = \mathbf{h}^m_{i,u} e^{\frac{jp\phi_{i,u}}{N}} x_{i,u} + \sum_{\substack{l=1 \\ l\neq u}}^{N_T} \mathbf{h}^m_{i,l} e^{\frac{jp\phi_{i,l}}{N}} x_{i,l} + \sum_{\substack{\forall k,l \\ k\neq i}} \mathbf{h}^m_{k,l} e^{\frac{jp\phi_{k,l}}{N}} x_{k,l} + \mathbf{n} \ , \quad (11)$$

where p is the discrete time index $p \in \{0, ..., n-1\}$. The random phase is defined in the range $0 \le \phi \le 2\pi/9$ having independent identically distributed (i.i.d.) properties for all BSs and transmit elements in the environment.

With these parameters, the evaluation for the achievable MSE is conducted again resulting in the performance given below. Fig. 5(a) indicates the achievable MSE in case of time variant channel conditions. It turns out that all



(a) Hadamard, fixed pattern (b) Hadamard, fixed pattern

Fig. 5. Normalized MSE obtained for the correlation estimator using the suggested virtual pilot pattern, in case of a dynamic channel, for the five strongest sectors (a) and the loss in the normalized MSE compared to static channel conditions (b).

estimation errors converge to almost the same value, i.e. ≈ -20 dB, for the maximum correlation length $n = 16$. Note that the estimation error of the strongest signal even increases from correlation length $n = 8$ to $n = 16$. In this case the error due to the phase rotation prevails the estimator gain. The loss in the normalized MSE compared to static channel conditions is given in Fig. 5(b). Again it turns out, that the estimation error basically increases for those channels with lowest MSE from the static case.

VI. Conclusion

The performance of multi-cell channel estimation based on virtual pilot sequences and partial correlation has been investigated. The scheme uses a specific cell planning for the sequences by which pilots of different sites are scrambled. Using block-orthogonal sequences, the less mobile users are the more channels, i.e. interferers can be estimated with sufficiently low mean square error. It is shown that multi-cell estimation clearly outperforms the covariance estimator required for interference rejection combining at the terminal. Also it is shown that the suggested scheme achieves a mean square error below -19 dB for the five strongest downlink signals received by a terminal. This is a good basis for advanced interference mitigation schemes using cooperative transmission. Future work will consider the throughput enhancement by such schemes based on realistic channel knowledge.

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Appendix A
Proof of scheme properties

For the cells a, b we write $a \equiv b$, if each base station a_1 has a correspondent base station b_1, whereas a MT placed in a measuring a_1 can expect the same channel estimation error like a MT placed in b measuring b_1. The error depends on the distances and pilot sequences of all BSs. More precisely, we write $a \equiv b$, if exists an bijective map $f_{ab} : C \times C \longrightarrow C \times C$, where C is the infinite set of cells. For arbitrary a_1, a_2 with $f_{ab}(a_1, a_2) = (b_1, b_2)$ the map must have the following properties: The distance between a, a_1 is equal to b, b_1. The same holds for a, a_2 and b, b_2. Also both pairs of pilot sequences assigned to a_1, a_2 and b_1, b_2 must have the same orthogonality property (true/false) after a correlation length of 2,4,8 and 16. In other words, each pair a_1, a_2 has a corespondent pair b_1, b_2 with the same distance and sequence (orthogonality) property. Regarding to our suggested scheme we define the following bijective map for any $a, b \in C$:

$$f_{ab}(a \oplus (r_1, d_1), a \oplus (r_2, d_2)) = (b \oplus (r_1, d_1), b \oplus (r_2, d_2)),$$

where r_1, d_1, r_2, d_2 are arbitrary integer numbers, $c \oplus (r, d)$ defines a new cell going r steps right and d steps down right along the hexagonal grid. The above mentioned distance property is fulfilled. To show the orthogonality property we checked only 16×15 pairs of sequences with a simulation program, because of the assignment repetition in the suggested scheme.

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